Problem List from the 36th Workshop in Geometric Topology

June 1, 2019

A word of caution: this was typed in real time and almost certainly has errors.

Guilbault - Does the Mazur compact contractible 4-manifold contain a pair of disjoint dunce hat spines? Does any compact contractible 4-manifold ($\neq B^4$) contain a pair of disjoint (dunce hat?) spines?

Ancel - How does the space of AM (approximate manifolds?) intersect approximate C spaces and approximate absolute neighborhood extensors?

Friedman - Suppose L is compact and $L \times \mathbb{R}$ is a manifold. Does there have to be an honest manifold such that $L \times \mathbb{R}$ is homeomorphic to $M \times \mathbb{R}$? This can't happen in high dimensions, but it's possible for dimension 3 and maybe 4.

Sunukjian - A 2-knot is an embedding of S^2 into S^4. You can define the equiv relation of concordance and cobordism. Concordance: take $S^4 \times I$, connect them in boundary. Cobordism: Use ANY 3 manifold to connect. Only one equiv class in both of these cases. But you can define $0-$ cobord or concordance to be those for which every level set is just a disjoint union of spheres. Fact: *Not all* 2-knots are 0-concordant. Question: Are all 2-knots 0-cobordant? If the answer is yes, then exotic S^4's cannot be optained via Gluck Twists.

Khan - GENERAL QUESTION Let G be a compact Hausdorff topological group. Let M be a $\mathbb{Z}$-Čech cohomology manifold (in the sense of Borel's 1960 "Seminar on Transformation Groups"), equipped with a G-action by homeomorphisms. Let K be a compact subset of M. Does K have only finitely many G-orbit types (i.e. G-conjugacy classes of stabilizers of points in K)?

AN IMPORTANT SPECIAL CASE Let T be a protorus (i.e. the inverse limit of toral groups), such as $T^I = (S^1)^I$ with the product topology for any set I. Let M be a $\mathbb{Z}$-cohomology manifold equipped with a T-action by homeomorphisms. Let K be a compact subset of M. Does K have only finitely many T-orbit types (in this case, only finitely many subgroups of T are stabilizers of points in K)?

REMARK The general question was proven true when G is a compact Lie group, by Floyd–Bredon in that seminar's book. For any abelian compact Hausdorff group G, the connected component of the identity is a protorus, and the quotient group is abelian, totally disconnected, and compact (e.g. the p-adic integers $\mathbb{Z}_p = \lim_{n\to\infty} \mathbb{Z}/p^n$, or more generally an abelian profinite group.)

Daverman - Does there exist a closed manifold that regularly covers itself with non-abelian covering group?

Mitra - Asymptotic Property C : A space X has APC if for every sequence of numbers, there is a natural number n s.t. X decomposes into n parts where each part is a union of R_i disjoint uniformly bounded sets. It is clear that finite asymptotic dimension implies APC. We know that dimension of the Higson corona is $\leq$ the adim, and it is also known that if adim is finite, these are equal. Question: if adim $= \infty$ and X has APC, is it true that the dimension of the Higson corona is infinite?